MA 161: Lesson 29

L'Hospital's Rule (4.7)

Indeterminate forms

Finding limits using L'Hopital's rule

Other indeterminate forms

Announcements

Exam 3 on Nov 20th - Lesson 18 (3.10) to Lesson 30 (4.9)

Study guide and instructions for exam 3 posted on brightspace

Office Hours

M, W, F: 245pm - 415pm

Evaluate the following limits

① $\lim_{x \to 2} \frac{x^2 - 2x}{x - 2} \rightsquigarrow \frac{0}{0}$

$\longrightarrow \lim_{x \to 2} \frac{x\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x \to 2} x = 2$

② $\lim_{x \to 2} \frac{x^2 - 2x}{(x-2)^2} \rightsquigarrow \frac{0}{0}$

$\longrightarrow \lim_{x \to 2} \frac{x(x-2)}{(x-2)^2} = \lim_{x \to 2} \frac{x}{x-2}$ } DNE

③ $\lim_{x \to \infty} \frac{x^2 - 2x}{x - 2} \rightsquigarrow \frac{\text{Large}}{\text{Large}}$ $\frac{\infty}{\infty}$

$\longrightarrow \lim_{x \to \infty} \frac{x\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x \to \infty} x = \infty$

$\frac{0}{0}$, $\frac{\infty}{\infty}$ $\rightsquigarrow$ indeterminate form.

L'Hopitals Rule

if $\lim_{x \to a} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

eg: #1 $\lim_{x \to 2} \frac{x^2-2x}{x-2} = \lim_{x \to 2} \frac{\frac{d}{dx}(x^2-2x)}{\frac{d}{dx}(x-2)} = \lim_{x \to 2} \frac{2x-2}{1} = \frac{2}{1}$

eg: #2 $\lim_{x \to 2} \frac{x^2-2x}{(x-2)^2} = \lim_{x \to 2} \frac{2x-2}{2(x-2)} = \frac{2}{0}$ Cannot Apply L'H.

$= DNE$

eg: $\lim_{x \to \infty} \frac{x^2-2x}{x-2} = \lim_{x \to \infty} \frac{2x-2}{1} = \infty$.

Eg: $\lim_{x \to 0} \frac{1-x}{\cos x}$

$$= \lim_{x \to 0} \frac{\frac{d}{dx}(1-x)}{\frac{d}{dx}(\cos x)} = \lim_{x \to 0} \frac{-1}{-\sin x} = \lim_{x \to 0} \frac{1}{\sin x}$$

$= DNE$

Cannot Apply L'H Rule Here

$$\lim_{x \to 0} \frac{1-x}{\cos x} = \frac{1-0}{\cos 0} = 1.$$

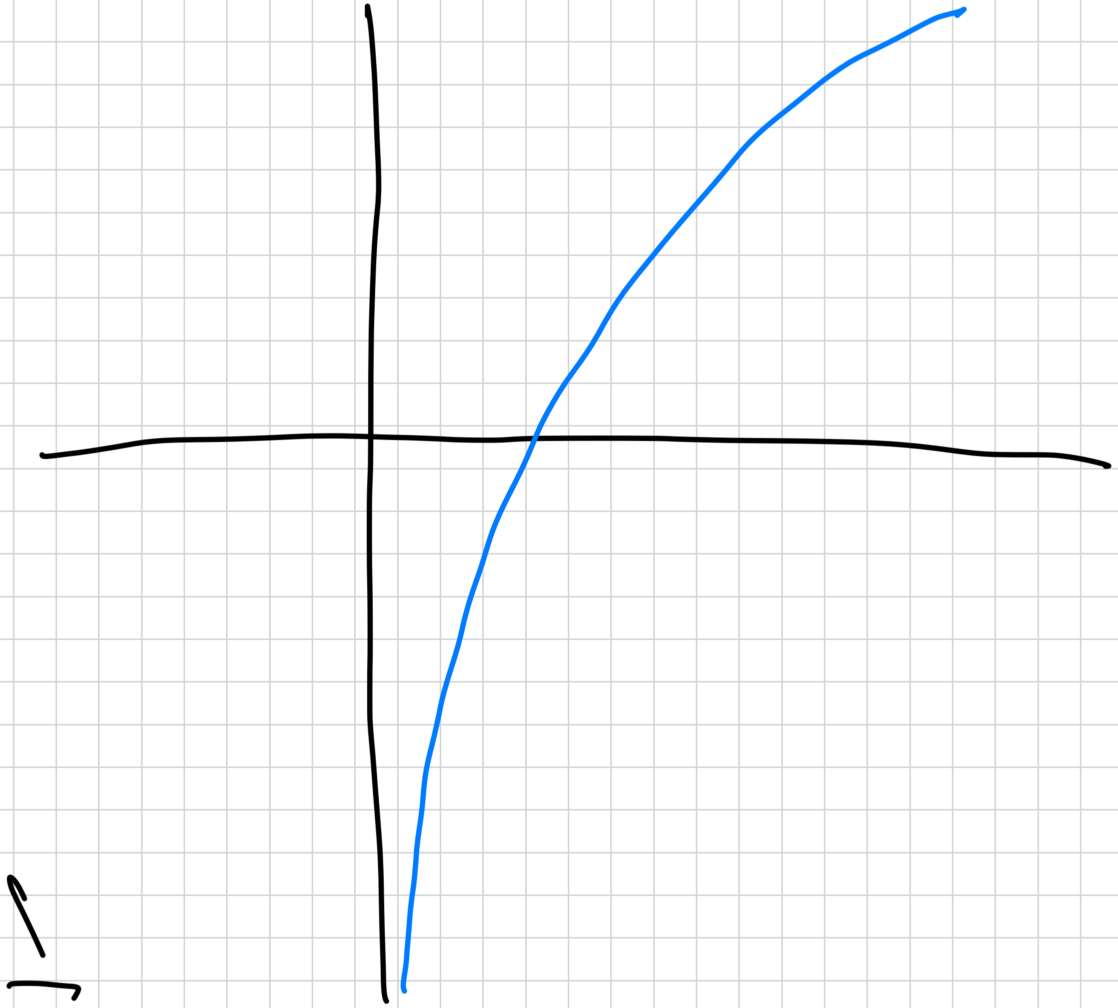
eg) #

$$\lim_{x \to \infty} \frac{\ln x}{x-1} \quad \left(\frac{\infty}{\infty}\right)$$

$$(H) = \lim_{x \to \infty} \frac{\frac{d}{dx}\ln x}{\frac{d}{dx}x-1}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1}{x}$$

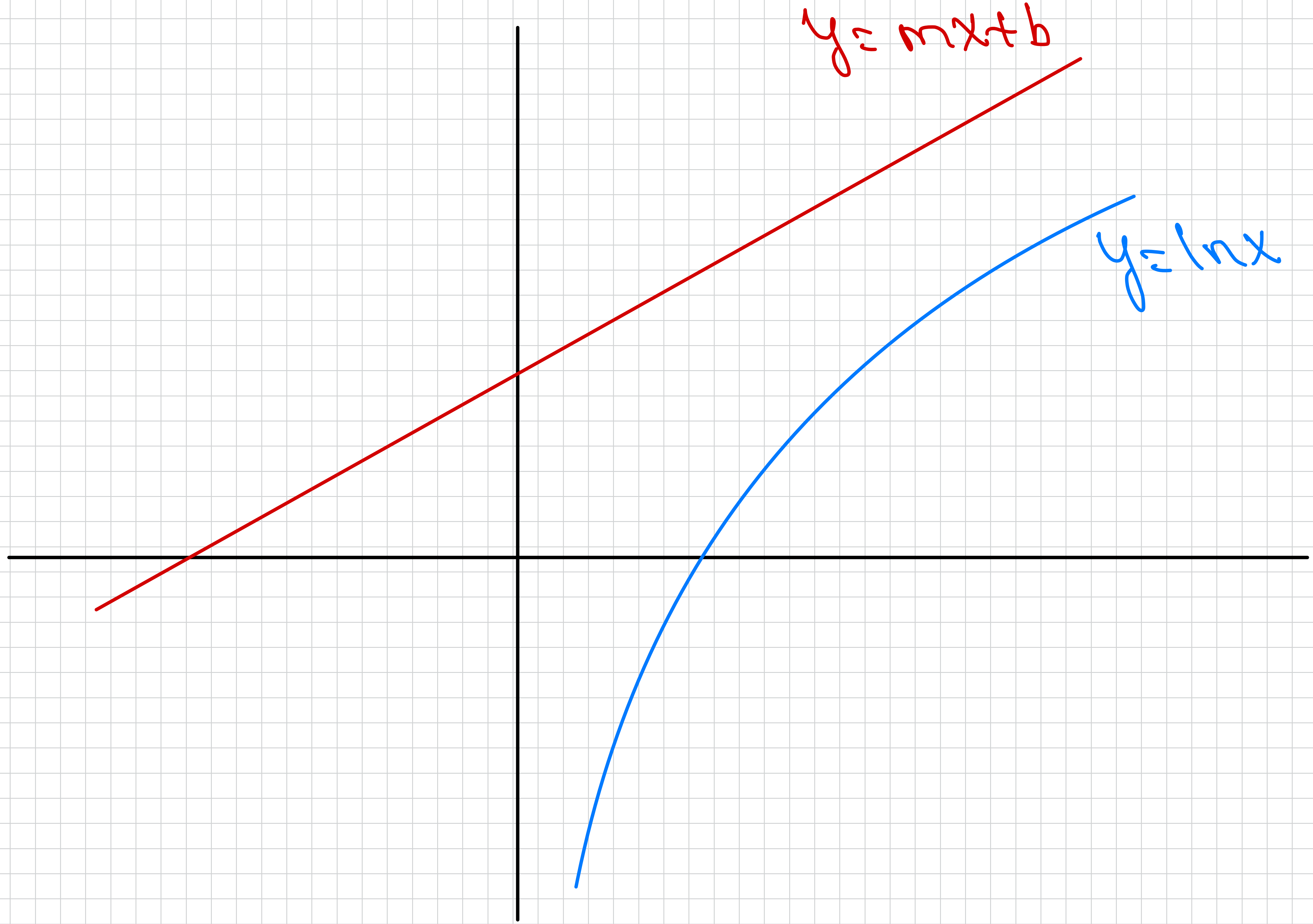
$$= 0$$



$$\lim_{x \to \infty} \frac{\ln x}{7x+5} = 0$$

Any Linear function! grows much faster than a logarithmic function

straight line



$y = mx + b$ grows faster than $y = \ln x$

eg: #

$$\lim_{x \to \infty} \frac{\ln x}{5x^3 - 7x + 3}$$

Cubic grows much faster than a St. line which grows faster than a logarithm.

LH

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{15x^2 - 7}$$

$$= \lim_{x \to \infty} \frac{1}{x(15x^2 - 7)} = \frac{1}{\text{LARGE}}$$

$$= 0.$$

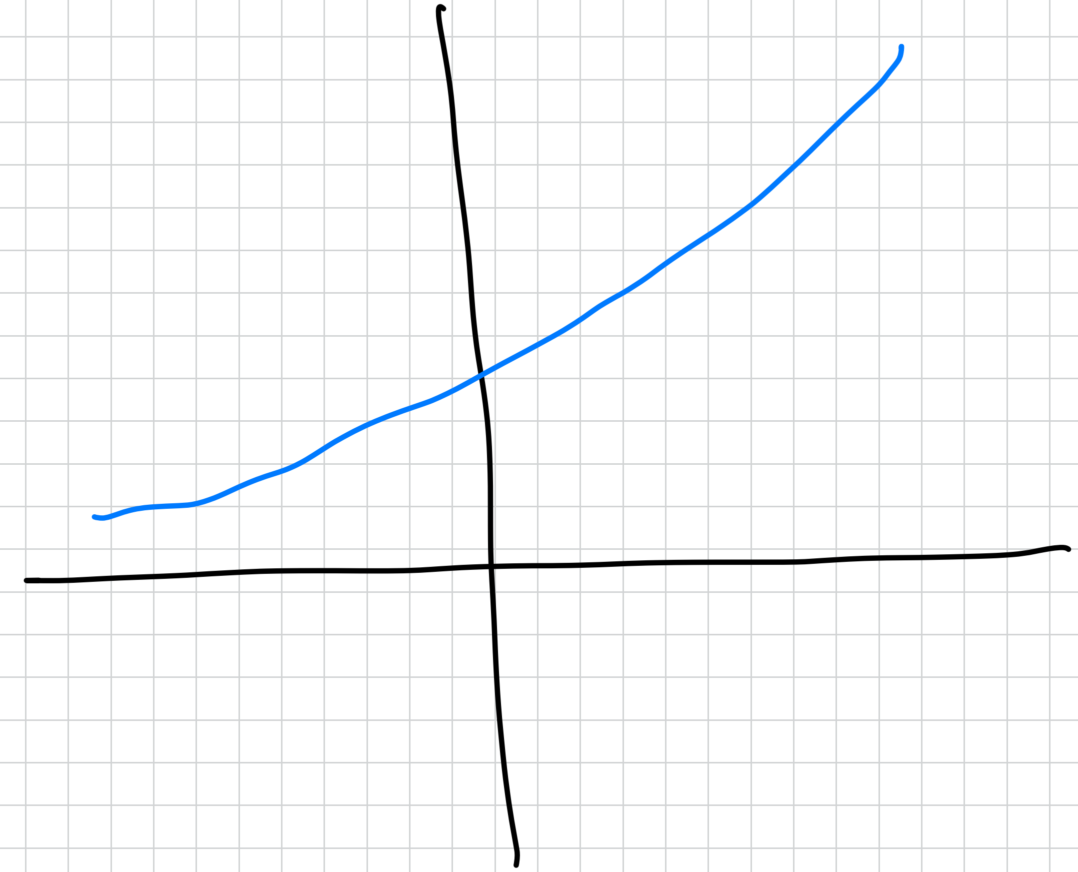
$= 0.$

Any Polynomial grow's faster than a logarithmic function.

eg:

$$\lim_{x\to\infty} \frac{x^5-12x^2+8}{e^x} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{5x^4-24x}{e^x} \quad \left(\frac{\infty}{\infty}\right)$$



$$= \lim_{x\to\infty} \frac{20x^3-24}{e^x} \quad \left(\frac{\infty}{\infty}\right)$$

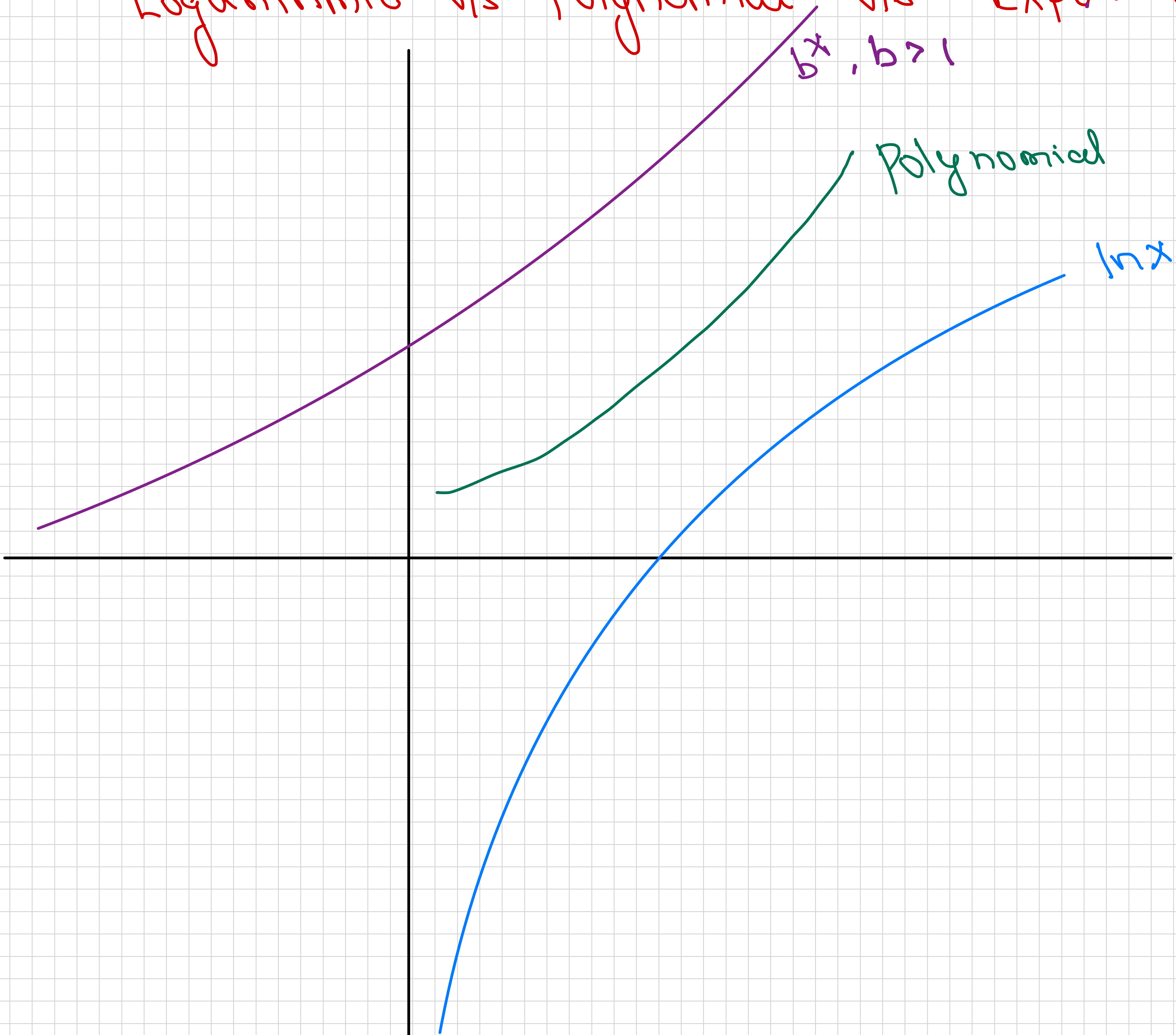
$$= \lim_{x\to\infty} \frac{60x^2}{e^x} \quad \left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty} \frac{120x}{e^x} = \lim_{x\to\infty} \frac{120}{e^x}$$

$$\frac{120}{\text{LARGE}}$$

$$= 0$$

exponential function grows Much faster than any polynomial.

Logarithmic v/s Polynomial v/s Exponential



eg: $\lim_{x\to\infty} \frac{x^2}{3^x}$ ⇝ $\frac{\text{poly.}}{\text{exponential}}$ ⇝ $\lim_{x\to\infty} \frac{x^2}{3^x} = 0$

eg: $\lim_{x\to 0} \frac{e^x}{\ln x + x^{98}}$ $x^{98} >>>> \ln x$

$\approx \lim_{x\to\infty} \frac{e^x}{x^{98}}$ $\frac{\text{exponential}}{\text{polynomial}}$

$= \infty$

What is $0 \cdot \infty$

$$\lim_{x \to \infty} x \cdot \frac{1}{x} = 1 \quad , \quad \lim_{x \to \infty} x^2 \cdot \frac{1}{x} = \infty$$

$\infty \cdot 0$ $\quad$ $\infty \cdot 0$

$\infty - \infty$

$$\lim_{x \to 0} \frac{1}{x} - \frac{1}{x} = 0 \quad , \quad \lim_{x \to \infty} x^2 - x = \infty$$

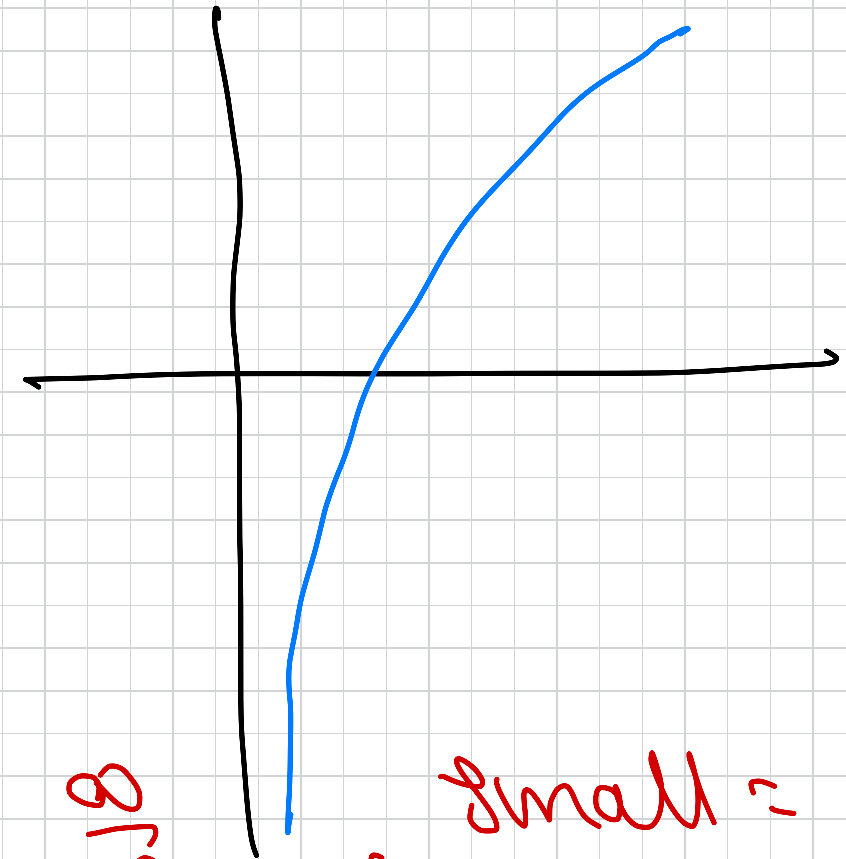
1^{∞}

$$\lim_{x \to 0^+} (0.99 + x)^{\frac{1}{x}} = 0 \quad , \quad \lim_{x \to 0^+} (1.01 + x)^{\frac{1}{x}} = \infty$$

$0 \cdot \infty$, $\infty - \infty$, 1^{∞} are also indeterminate forms eg: $\lim_{x\to0^+} x\ln x$ $\rightsquigarrow$ $\underbrace{0\cdot\infty}$

try to use $f(x) = \frac{1}{(1/f(x))}$

and convert it to $\frac{0}{0}$ or $\frac{\infty}{\infty}$



small $= \frac{1}{\text{Large}}$

$$= \lim_{x\to0^+} \frac{\ln x}{(1/x)} \qquad \frac{\infty}{\infty}$$

$$\text{L'H} = \lim_{x\to0^+} \frac{(1/x)}{(-1/x^2)} = \lim_{x\to0^+} \frac{-x^2}{x} = \lim_{x\to0^+} -x = 0.$$

eg! $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$ $\infty - \infty$ make it one fraction

$$= \lim_{x \to 0^+} \frac{e^x - 1 - x}{(x)(e^x - 1)} \quad \left(\frac{0}{0}\right)$$

$$\text{L'H} = \lim_{x \to 0^+} \frac{e^x - 1}{e^x - 1 + xe^x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 0^+} \frac{e^x}{e^x + e^x + xe^x} = \frac{1}{1 + 1 + 0} = \frac{1}{2}$$

eg #:

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x$$

1^∞ take ln & then try to get to $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$L = \lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x$$

$$\ln L = \lim_{x\to\infty} \ln\left(\left(1+\frac{3}{x}\right)^x\right) = \lim_{x\to\infty} \underbrace{x \cdot \ln\left(1+\frac{3}{x}\right)}_{\infty \cdot 0}$$

$$= \lim_{x\to\infty} \frac{\ln(1+3/x)}{(1/x)} \quad \frac{0}{0}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+3/x}\cdot\frac{-3}{x^2}}{\frac{-1}{x^2}} = 3.$$

take exponential on both sides

$$\ln L = 3$$

$$e^{\ln L} = e^3 \longrightarrow L = \lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x = e^3.$$